7308

Subpoint Prediction for Direct Readout Meteorological Satellites

L. E. EBER[1]

ABSTRACT

The National Environmental Satellite Service (NESS) provides orbital information on meteorological satellites with direct transmission systems, through APT (Automatic Picture Transmission) Predict messages sent over standard weather communications networks. With periodic access to this information, operators of independent APT ground receiving stations can extrapolate, by means of nodal period and nodal increment, to determine future orbits within receiving range of their station. A technique for the prediction of subpoint location along an orbit as a function of time after ascending node was developed from consideration of Kepler's laws and derived expressions for the force due to the earth's gravitational potential. Subpoint latitudes and longitudes computed by this technique are within 0.1 degree of those given in NESS predictions.

INTRODUCTION

Users of APT (Automatic Picture Transmission) and Direct Readout Scanning Radiometer data from meteorological satellites, who operate APT ground receiving stations, need orbital information for scheduling transmission pickup and for tracking and locating subpoints for data identification. Such information is provided by the National Environmental Satellite Service (NESS) through APT Predict messages transmitted over meteorological teletype circuits.[2] In cases where these messages are not regularly available, or where advance scheduling is desired, the dates, times, and longitudes of the ascending nodes for selected orbits can be extrapolated by means of nodal period and nodal increment as much as a month ahead with acceptable accuracy. Subpoint locations at selected time invervals along the track also can be predicted for future orbits if certain other orbital characteristics are known. These include the semimajor axis and eccentricity of the orbital ellipse, the orbital inclination, and the geocentric angle of the perigee measured, at a known reference time, from the ascending node in the orbital plane.

UNDISTURBED ORBITAL MOTION

The orbit of a meteorological satellite is an ellipse with the earth's center at one focus. According to Kepler's laws, the motion of a satellite is such that a ray from the focus to the satellite sweeps out equal areas in equal times. Consequently the angular velocity of the satellite is greatest at perigee when the satellite is at its nearest approach to the earth and least at apogee when it is farthest away. The area swept out per unit time by the satellite can be expressed by:

$$\frac{R^2 d\ \alpha}{2dt} = \frac{\pi AB}{P}$$

where $d\ \alpha/dt$ is the angular velocity, R is the distance from the center of the earth to the satellite, A and B are, respectively, the semimajor and semiminor axes of the orbital ellipse, and P is the orbital period. Rearrangement of terms

[1] Southwest Fisheries Center, National Marine Fisheries Service, NOAA, La Jolla, CA 92037.

[2] ESSA (Environmental Science Services Administration) Direct Transmission System Users Guide, prepared by the National Environmental Satellite Center, U.S. Department of Commerce, 1969.

gives an explicit equation for the angular velocity, as follows:

$$\frac{d\alpha}{dt} = \frac{2\pi AB}{PR^2}. \qquad (1)$$

The values of A and B are related to the eccentricity, e, of an ellipse by the expression

$$e = \frac{F}{A}$$

where

$$F = \sqrt{A^2 - B^2}$$

and represents the distance from the center to either focus of the ellipse.

The relation between R and α can be obtained from the basic equation of an ellipse in polar coordinates, with the origin at one focus and the polar axis coincident with the major axis of the ellipse:

$$R = \frac{A\ (1 - e^2)}{1 \pm e \cos \alpha}. \qquad (2)$$

DISTURBED ORBITAL MOTION

Because the earth is not a concentrically homogeneous sphere, but rather an oblate ellipsoid of revolution, the angular velocity is not exactly defined by Equation (1). The elliptic motion is governed by a force which can be defined in terms of the Newtonian gravitational potential U, as follows (Menzel, 1960):

$$U = \frac{K^2\ M}{R} + \frac{K^2\ (I - J)\ (1 - 3\sin^2\phi)}{2R^3} \qquad (3)$$

where M is the mass of the earth,
- K^2 is the constant of gravitation,
- I is the moment of inertia about the polar axis,
- J is the moment of inertia about any diameter in the earth's equatorial plane, and
- ϕ is the angle between the equatorial plane and a line from the earth's center to the satellite (i.e., ϕ = latitude).

The expression $(I - J)$ can be related to the earth's mass M, the equatorial radius a, the earth's flattening f, and the ratio q of the earth's centripetal acceleration at the equator to gravity, as follows:

$$I - J = (2/3)\ Ma^2\ (f - q/2). \qquad (4)$$

Substituting Equation (4) in Equation (3), we get

$$U = \frac{K^2\ M}{R} + \frac{K^2\ Ma^2\ (f - q/2)\ (1 - 3\sin^2\phi)}{3R^3}. \qquad (5)$$

A more precise form of expression for the earth's gravitational potential is given in terms of spherical harmonics by

$$U = \frac{K^2\ M}{R} - \frac{K^2\ M}{R} \sum_{n=2}^{\infty} J_n \left(\frac{a}{R}\right)^n P_n\ (\sin\phi)$$

where the J_n are constants which can be determined from observations of earth satellites, and the P_n $(\sin\phi)$ are Legendre polynomials (Runcorn, 1967). The first term in the series (corresponding to $n = 2$) represents the main effect of the earth's oblateness. The value of J_2 has been determined as 1.0826×10^{-3}, which is 400 times greater than J for any of the higher n. The Legendre polynomial for $n = 2$ is

$$P_2\ (\sin\phi) = (3/2)\sin^2\phi - 1/2.$$

If terms in higher n are omitted, the expression for gravitational potential becomes

$$U = \frac{K^2\ M}{R} + \frac{K^2\ M\ \ a^2\ J_2\ \ (1 - 3\sin^2\phi)}{2R^3}.$$

This expression is equivalent to Equation (5) if terms of second and higher order are omitted in the following relation between J_2 and the constants f and q:

$$J_2 = (2/3)\ (f - q/2) - (1/3)\ f^2 + (1/2)q^2 + (2/21)fq.$$

The values of f and q have been determined as 3.353×10^{-3} and 3.468×10^{-3}, respectively, so that

$$(2/3)\ (f - q/2) = 1.0794 \times 10^{-3}.$$

The value of $K^2 M$ is 3.986×10^5 km^3 sec^{-2}.

The gravitational acceleration is obtained by differentiating (5) with respect to R to get

$$\frac{dU}{dR} = \frac{-K^2 M}{R^2} - \frac{K^2 Ma^2 (f - q/2)(1 - 3 \sin^2 \phi)}{R^4}. \quad (6)$$

This expression for gravitational acceleration due to the mass of the earth could be equated directly to the centripetal acceleration of an earth satellite if it were traveling in a true circular orbit. In an elliptical orbit, however, there is a deviation from circular motion which can be identified by substituting into (1) the following expression from Kepler's third law for the orbital period:

$$P = \frac{2\pi A^{3/2}}{K\sqrt{M}}.$$

The substitution gives

$$\frac{d\alpha}{dt} = \frac{KAB}{R^2}\sqrt{\frac{M}{A^3}}, \quad (7)$$

from which we obtain the following equation for centripetal acceleration:

$$R\left(\frac{d\alpha}{dt}\right)^2 = \frac{K^2 M B^2}{R^3 A}. \quad (8)$$

The right side of Equation (8) can be considered as the product of (B^2/RA) and the quantity (K^2M/R^2). The latter is equivalent, except in sign, to the first term on the right side of Equation (6) and represents the gravitational acceleration for undisturbed motion. The correction for the ellipticity of an orbit is contained in the factor B^2/RA. Its value approaches 1 when an orbit is nearly circular. By applying this factor to the right side of Equation (6) we can relate the earth's gravitational acceleration to the centripetal acceleration of an earth satellite as follows:

$$R\left(\frac{d\alpha}{dt}\right)^2 = \frac{B^2}{RA}\left(\frac{K^2M}{R^2} + \frac{K^2Ma^2 (f - q/2)(1 - 3\sin^2\phi)}{R^4}\right),$$

and the angular motion is

$$\frac{d\alpha}{dt} = \frac{B}{R}\left(\frac{K^2M}{R^2A} + \frac{K^2M a^2 (f - q/2)(1 - 3\sin^2\phi)}{R^4A}\right)^{1/2}. \quad (9)$$

SUBPOINT LOCATION

Satellite orbit predictions are given in terms of subpoint locations at fixed intervals of time following the northward equator crossing, referred to as the ascending node. Numerical integration of Equation (9), carried out with suitably small time increments, can be made to yield successive values of α requisite for subpoint computation. If α is taken to be zero when the satellite is at perigee, then the plus sign preceding $e \cos \alpha$ in Equation (2) is applicable and R is given by

$$R = \frac{A(1 - e^2)}{1 + e\cos\alpha}. \quad (10)$$

R can be computed from Equation (10) for each iteration in the numerical integration of Equation (9) using the value of α from the preceding iteration increased by one-half the average angular displacement $[360(\Delta t)/2P]$ during the selected time increment, Δt. In order to start the numerical integration of Equation (9) at the ascending node, an initial value of α must be determined from the orientation of the orbital ellipse. The latter is expressed as the argument of perigee, which is the geocentric angle of the perigee measured in the orbital plane from the ascending node in the direction of motion. The

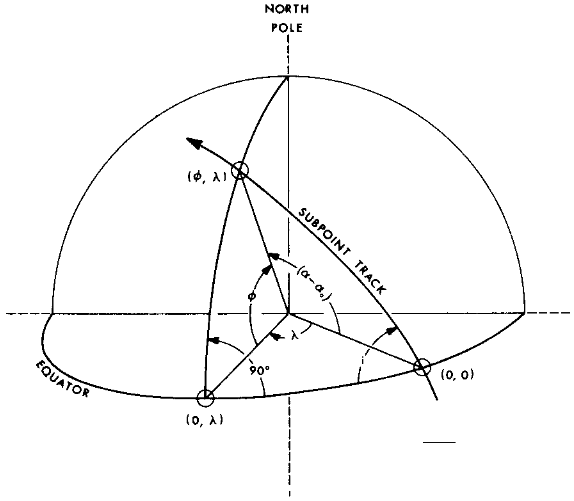

Figure 1.—Right spherical triangle formed by the equator, the orbital subpoint track and the meridian through the subpoint (ϕ, λ).

initial value of α is obtained by subtracting the argument of perigee from 360°.

The gravity effect of the earth's equatorial bulge causes the perigee to shift slightly during each orbit. Consequently, it is necessary to know the rate at which the orbital ellipse is shifting in order to determine the correct initial values of α for prediction of future orbits.

The locus of orbital subpoints for an earth satellite traces a great circle on a sphere concentric with the earth. Each subpoint can be represented by a pair of coordinates (ϕ, λ), where ϕ is the geocentric latitude and λ is the longitudinal displacement of the subpoint measured from the longitude of the ascending node. Subpoint coordinates corresponding to any point of the orbit can be determined from its angular displacement, measured along the orbital track from the ascending node. If α is the angular coordinate (measured from perigee, as defined in the preceding section) and α_0 is the value of α at the ascending node, the angular displacement is $(\alpha - \alpha_0)$. Figure 1 shows these quantities as components of a right spherical triangle whose vertices are defined by the subpoint (ϕ, λ), the coordinates of the ascending node (0, 0) and the point $(0, \lambda)$. The two sides forming the right angle are ϕ and λ respectively. The opposite side is $(\alpha - \alpha_0)$ and is a segment of the subpoint track. The angle opposite ϕ, denoted by i, is the inclination of the orbital plane to the equatorial plane. Application of the trigonometric formulas derived from Napier's rules yields the following relationships:

$$\sin \phi = \sin(i)\sin(\alpha - \alpha_0), \tag{11}$$

$$\tan \lambda = \cos(i)\tan(\alpha - \alpha_0), \text{ and} \tag{12}$$

$$\tan \phi = \tan(i)\sin \lambda. \tag{13}$$

Equations (11) and (12) give explicit formulas for ϕ and λ as a function of (i) and $(\alpha - \alpha_0)$. Equation (13) represents the great circle traced by the orbital subpoints.

ORBIT PREDICTION

The concepts developed in the preceding sections deal with the computation of satellite subpoints relative to the time and longitude of the ascending node. Application of these concepts to advance preparation of orbit schedules requires, therefore, prediction of the times and longitudes of ascending nodes for future orbits. The APT Predict messages give orbital period to the nearest second and longitudinal displacement per orbit to the nearest hundredth of a degree. This precision is adequate to extrapolate a few orbits ahead. Greater precision is needed for satisfactory extrapolation beyond a hundred orbits and can be obtained from NESS or can be gained by empirical adjustments.

The orbital elements necessary for computation of subpoint locations for extrapolated orbits, also obtainable from NESS, include the length of the semimajor axis, eccentricity, orbital inclination, argument of perigee at a known time or reference orbit, and the rate of change of perigee. The latter is equal to the rate of change of α_0, the value of the angular displacement α at the ascending node, which can be computed from the following expression (Runcorn, 1967):

$$d\,\alpha_0/dt = 4.98(a/A)^{3.5}\,(1 - e^2)^{-2}\,(5\cos^2 i - 1)$$
(degrees per day),

where e is the eccentricity of the orbital ellipse,
i is the inclination of the orbital plane to the earth's equatorial plane,
a is the earth's equatorial radius, and
A is the semimajor axis of the ellipse, as previously defined.

For the satellite ESSA-8, the value of i is 78.3°, $A = 7815.37$ km, and $e = 0.00323$. With these values and taking 6378 km for a, we compute

$$\frac{d\,\alpha_0}{dt} = -1.943 \text{ deg per day.}$$

Given the nodal period of ESSA-8 as 115.703 min we find, alternately, that

$$\frac{d\,\alpha_0}{dt} = -0.15476 \text{ deg per orbit.}$$

The minus sign indicates that the change is opposite to the angular motion of the satellite.

Having determined α_0 for a selected orbit and starting at the ascending node where $\alpha = \alpha_0$, the radial distance R can be computed from Equation (10) for use in the numerical integration of Equation (9), to yield the angular displacement $(\alpha - \alpha_0)$ for successive intervals of desired length after the time of ascending node. The values of ϕ and λ can then be obtained with Equations (11) and (12). These are the spherical coordinates of the point of the orbit relative to a nonrotating frame of reference with origin at the earth's center.

In order to plot the subpoint track on a chart, the spherical coordinates of points on the orbit need to be transformed to the geographic (geodetic) latitudes and longitudes of the corresponding subpoints. The latter are the locations on the earth's surface where the local vertical passes through the satellite. The angle which local vertical makes with the equatorial plane defines the geodetic latitude of the subpoint (Bowditch, 1958). Owing to the earth's ellipsoidal shape, a line from the satellite to the center of the earth intersects the surface at a point north of the satellite subpoint. The geodetic latitude at this point of intersection is a close approximation to that of the subpoint and can be computed from the following expression:

$$\text{geodetic latitude} = \text{arc ctn}\,[(1 - q)\,\text{ctn}\,\phi]$$

where q is the ratio of centripetal acceleration at the equator to gravity, as defined earlier. This expression is derived from the vector difference between the true force of gravitation on a unit mass at a point on the earth and the apparent force of gravity at that point, as a result of the earth's rotation. Taking

$$q = 3.468 \times 10^{-3}$$

the maximum correction for geodetic latitude computed with the above expression is about 0.1 degree at latitude 45 degrees.

The longitude of the satellite subpoint can be obtained by adding the longitude of the ascending node to the value of λ computed with Equation (12). Adjustment for the earth's rotation is made by adding to this sum the product of the rate of rotation (0.25 degree of longitude per minute) and the elapsed time from the ascending node to the point of the orbit for which the subpoint location is desired. A correction must be made for the precession of the orbital plane which, in a sun-synchronous orbit, amounts to 360 degrees in 365 days and is subtractive.

APPLICATION

The foregoing procedures have been implemented in a computer program for preparing orbit schedules at the Southwest Fisheries Center, La Jolla Laboratory, National Marine Fisheries Service. The program includes a section for each active satellite in which appropriate values are assigned for semimajor axis, eccentricity, inclination, anomalistic period, argument of perigee at a known reference orbit and the rate of change of the argument of perigee. All of these quantities are taken directly from information sheets furnished by NESS.

In running the program, a data card is submitted for each schedule to be printed. On it are punched the name of the satellite and the orbit number, day, hour, minute, second, and longitude of the ascending node for a particular orbit as read from a recent or current APT Predict message. The card is also punched with the standard (or daylight) time zone at the station for conversion from universal to local time, and a specification of the number of minutes after ascending node to which computed orbit reference times will apply. The latter tells the APT ground station operator when to expect to begin receiving transmission. The number specified indicates to the program whether orbits selected for subpoint computation are to be determined by south-to-north or north-to-south traverses over the station.

ORBIT SCHEDULE FOR ESSA-8 OCT 21-28

INITIAL DATA: 17637 20 13 31 32 22628 8

ORBIT REFERENCE TIMES ARE 36 MINUTES AAN (PST)

SUBPOINT LOCATIONS AT ONE MINUTE INTERVALS

	MIN	LAT	LON	MIN	LAT	LON	MIN	LAT	LON	MIN	LAT	LON	MIN	LAT	LON
DATE: 21 ORBIT17651	37	60.95	82.59	38	58.06	85.32	39	55.13	87.67	40	52.17	89.74	41	49.18	91.58
HR 8 MIN 53 SEC 23	42	46.18	93.23	43	43.15	94.74	44	40.12	96.13	45	37.08	97.43	46	34.02	98.64
LONG ASC NODE: 84.82E	47	30.96	99.78	48	27.90	100.87	49	24.83	101.92	50	21.75	102.92	51	18.68	103.99
	52	15.60	104.84	53	12.52	105.77	54	9.43	106.68	55	6.35	107.58	56	3.27	108.47
DATE: 21 ORBIT17652	37	60.95	111.27	38	58.06	113.99	39	55.13	116.35	40	52.17	118.41	41	49.18	120.25
HR 10 MIN 48 SEC 5	42	46.18	121.91	43	43.15	123.42	44	40.12	124.81	45	37.08	126.10	46	34.02	127.31
LONG ASC NODE: 56.14E	47	30.96	128.46	48	27.90	129.55	49	24.83	130.59	50	21.75	131.60	51	18.68	132.57
	52	15.60	133.52	53	12.52	134.44	54	9.43	135.35	55	6.35	136.25	56	3.27	137.14
DATE: 22 ORBIT17664	37	60.94	95.38	38	58.06	98.11	39	55.13	100.46	40	52.17	102.52	41	49.18	104.36
HR 9 MIN 44 SEC 31	42	46.17	106.02	43	43.15	107.53	44	40.12	108.92	45	37.08	110.21	46	34.02	111.42
LONG ASC NODE: 72.03E	47	30.97	112.57	48	27.90	113.66	49	24.83	114.70	50	21.76	115.71	51	18.68	116.68
	52	15.60	117.63	53	12.52	118.55	54	9.44	119.46	55	6.36	120.36	56	3.28	121.25
DATE: 23 ORBIT17676	37	60.94	79.49	38	58.05	82.22	39	55.12	84.57	40	52.16	86.63	41	49.18	88.47
HR 8 MIN 40 SEC 57	42	46.17	90.13	43	43.15	91.64	44	40.12	93.03	45	37.08	94.32	46	34.03	95.53
LONG ASC NODE: 87.92E	47	30.97	96.68	48	27.90	97.77	49	24.83	98.81	50	21.76	99.81	51	18.69	100.79
	52	15.61	101.73	53	12.53	102.66	54	9.45	103.57	55	6.37	104.47	56	3.28	105.36
DATE: 23 ORBIT17677	37	60.94	108.17	38	58.05	110.89	39	55.12	113.25	40	52.16	115.31	41	49.18	117.15
HR 10 MIN 35 SEC 39	42	46.17	118.80	43	43.15	120.31	44	40.12	121.70	45	37.08	123.00	46	34.03	124.21
LONG ASC NODE: 59.25E	47	30.97	125.35	48	27.90	126.44	49	24.83	127.49	50	21.76	128.49	51	18.69	129.46
	52	15.61	130.41	53	12.53	131.34	54	9.45	132.25	55	6.37	133.14	56	3.29	134.03
DATE: 24 ORBIT17689	37	60.94	92.28	38	58.05	95.00	39	55.12	97.36	40	52.16	99.42	41	49.18	101.26
HR 9 MIN 32 SEC 6	42	46.17	102.91	43	43.15	104.42	44	40.12	105.81	45	37.08	107.10	46	34.03	108.32
LONG ASC NODE: 75.14E	47	30.97	109.46	48	27.91	110.55	49	24.84	111.59	50	21.77	112.60	51	18.69	113.57
	52	15.61	114.52	53	12.54	115.44	54	9.46	116.35	55	6.37	117.25	56	3.29	118.14
DATE: 24 ORBIT17690	37	60.94	120.96	38	58.05	123.68	39	55.12	126.03	40	52.16	128.10	41	49.18	129.93
HR 11 MIN 26 SEC 48	42	46.17	131.59	43	43.15	133.10	44	40.12	134.49	45	37.08	135.78	46	34.03	136.99
LONG ASC NODE: 46.46E	47	30.97	138.14	48	27.91	139.23	49	24.84	140.27	50	21.77	141.27	51	18.69	142.25
	52	15.61	143.19	53	12.54	144.12	54	9.46	145.03	55	6.38	145.93	56	3.29	146.82
DATE: 25 ORBIT17702	37	60.94	105.07	38	58.05	107.79	39	55.12	110.14	40	52.16	112.21	41	49.18	114.04
HR 10 MIN 23 SEC 14	42	46.17	115.70	43	43.15	117.21	44	40.12	118.60	45	37.08	119.89	46	34.03	121.10
LONG ASC NODE: 62.35E	47	30.97	122.25	48	27.91	123.33	49	24.84	124.38	50	21.77	125.38	51	18.70	126.35
	52	15.62	127.30	53	12.54	128.23	54	9.46	129.14	55	6.38	130.03	56	3.30	130.92

Figure 2.—Example of computer generated satellite orbit schedule with subpoint locations for selected orbits.

TABLE 1.--Verification of computed subpoint locations for ESSA-8. Computed values minus APT Predict values are given in degrees latitude or longitude.

Min AAN*	Approximate latitude	Orbit 15239 (4/12/72)		Orbit 15641 (5/14/72)		Orbit 16030 (6/14/72)		Orbit 16444 (7/17/72)		Orbit 16846 (8/18/72)		Orbit 17235 (9/18/72)		Orbit 17637 (10/20/72)	
		Lat	Long	Lat	Long	Lat	Long	Lat	Long	Lat	Long	Lat	Long	Lat	Long
34	70°	.05	.00	.04	-.01	.06	.00	-.01	-.02	.04	-.02	.00	.06	.00	-.03
36	64°	.03	-.03	.06	.02	.05	-.01	.01	-.01	.03	.00	-.02	-.02	.09	.00
38	59°	.00	.03	.06	-.04	.01	-.01	-.01	.07	-.03	-.02	.00	-.01	.06	-.05
40	53°	.01	.06	.01	-.01	-.01	.01	-.02	.00	.01	.00	.01	.01	.07	-.03
42	47°	.01	.07	.05	-.02	-.04	.07	.05	.03	.04	.02	.00	.06	.08	-.03
44	41°	.05	-.03	.03	-.01	-.03	.01	.04	.02	-.01	.01	.03	.00	.02	-.03
46	35°	.04	-.02	.06	-.01	.03	.05	-.01	-.01	.00	-.02	.01	.04	.02	-.02
48	29°	.01	.02	.07	.03	.06	.00	-.01	.07	.06	.06	.05	.00	.10	.01
50	22°	.05	-.03	.06	-.02	.06	.07	.07	.05	-.01	.04	.08	.06	.05	.06
52	16°	.09	-.01	.03	.00	.05	.00	.02	-.01	.01	-.02	.09	.00	.09	.02
54	10°	.12	.03	.10	.03	.02	.04	.06	.04	.01	.04	.09	.04	.03	.02
56	4°	.04	.02	.06	.01	.08	.02	.08	.04	.00	.04	.09	.04	.06	.01

*Minutes after ascending node.

A sample schedule, computed and printed for ESSA-8, is shown in Figure 2. ESSA-8 transmits only during the daytime, when it is following a north-to-south track. Accordingly, the reference time appearing to the left of each set of subpoint tabulations is 36 min later than the time of ascending node for the orbit indicated. The numbers labeled "INITIAL DATA" were read from the data card and are printed for verification. The subpoint locations are printed to the nearest hundredth of a degree of latitude or longitude, for each minute from 37 to 56 min after ascending node, which covers the daytime track from about lat. 60° N to lat. 4° N. The subpoint location corresponding to a picture transmitted by the satellite can be found by interpolation, based on determination of the time of the picture relative to the time of ascending node. The schedule includes only those orbits within range of the APT ground station at the La Jolla Laboratory.

A comparison of computed subpoint locations with those given in APT Predict messages for seven orbits of ESSA-8, completed between April and October 1972, is presented in Table 1. Subpoint locations are given to the nearest tenth of a degree in APT Predict messages, while computed values were carried to the nearest hundredth of a degree. Differences between computed values and corresponding APT Predict values averaged less than 0.06 degree of latitude or longitude. Since ESSA-8 completes an orbit in a little less than 2 hr, the motion of the satellite subpoint is about 3 nautical miles per second. An error of 0.05 degree latitude (equal to 3 nautical miles) in subpoint location is equivalent, therefore, to an error of about 1 sec in determining the time of an observation from the satellite.

LITERATURE CITED

BOWDITCH, N.
1958. American practical navigator an epitome of navigation. U.S. Navy Hydrogr. Off. Wash., D. C., 1524 p.
MENZEL, D. H. (editor).
1960. Fundamental formulas of physics. Vol. 2. Dover Publ., Inc., N. Y., p. 680-690.
RUNCORN, S. K., (editor-in-chief).
1967. International dictionary of geophysics. Vol. 7. Permagon Press, Lond., p. 80-82, 326-331.