Appendix B4: Envelope Method

Stock assessment models typically incorporate two primary sources of information: estimates of total catch (landings plus discards), and fishery-independent indices of abundance. The former quantities provide estimates of population scale, the latter quantities provide measures of trend. Total catch provides some insight into the scale of the population but without additional information it is impossible to determine if total catch is the result of a low fishing mortality rate applied to a large population or a high fishing mortality rate applied to a small population. Fishery independent stock size estimates from trawl surveys, expressed in terms of average catch per tow, approximate the true population size subject to an arbitrary scalar that reflects gear efficiency, availability, and the variability in the realization of the sampling design. Collectively these factors are called catchability and denoted as the parameter q.

The uncertainty in the interpretation of these two basic quantities is addressed explicitly in an assessment model but the underlying relationships can be obscured by complexity of the mathematics and tradeoffs among poorly estimated parameters. Here we propose a simple approach to reconcile these perspectives on stock size that provides a feasible range or "envelope" of population sizes. The purpose of this exercise is not to replace the delay-difference model used in this assessment. Instead the purpose is to demonstrate that the assessment model is consistent with the implications simpler measures of stock size.

Let I_t represent the observed index of biomass at time t and C_t represent the catch at time t. The estimated total biomass consistent with the index is

$$B_t = \frac{I_t}{q} \qquad (1)$$

where q is an assumed value. The biomass consistent with observed catch can be obtained from the catch equation as

$$B_t = \frac{C_t}{\frac{F}{F+M}\left(1-e^{-(F+M)}\right)} \qquad (2)$$

where F is unknown. Thus biomass can be written as a function of arbitrary scalars q and F. These equations can be generalized and written as

$$\begin{aligned} \hat{B}_{1,t} &= B(I_t, q_{Low}) \\ \hat{B}_{2,t} &= B(I_t, q_{High}) \\ \hat{B}_{3,t} &= B'(C_t, F_{Low}, M) \\ \hat{B}_{4,t} &= B'(C_t, F_{High}, M) \end{aligned} \qquad (3)$$

In theory the above measures of stock biomass should be consistent. Prior information on the suitable range for q can be obtained from analyses of relative survey catchability as detailed in the main body of the report. The suitable range of F values can

obtained from analogy with other fisheries, or more simply by picking a wide range of values.

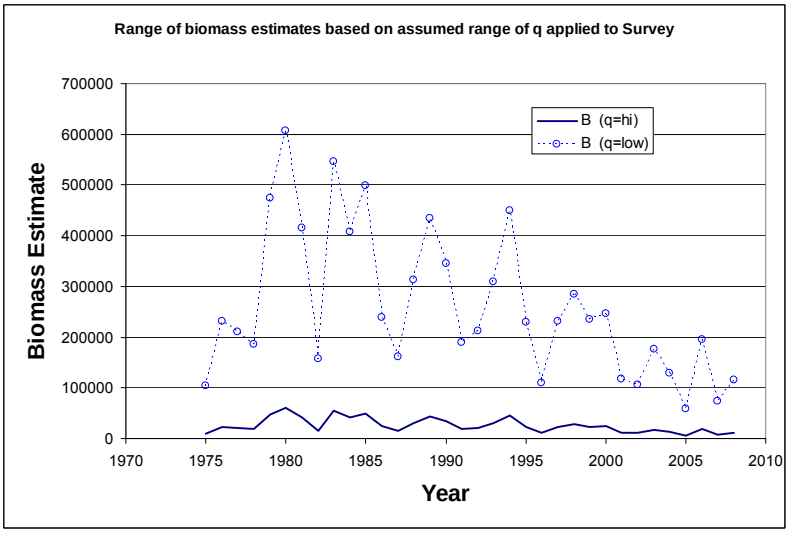
By inspection it is evident that $B_{1,t}$ and $B_{3,t}$ constitute an upper range, and $B_{2,t}$ and $B_{4,t}$ constitute a lower range. Upper and lower bounds consistent with these estimates are

$$\begin{aligned} \hat{B}_{upper,t} &= \min(B_{1,t}, B_{3,t}) \\ \hat{B}_{lower,t} &= \max(B_{2,t}, B_{4,t}) \end{aligned} \qquad (4)$$

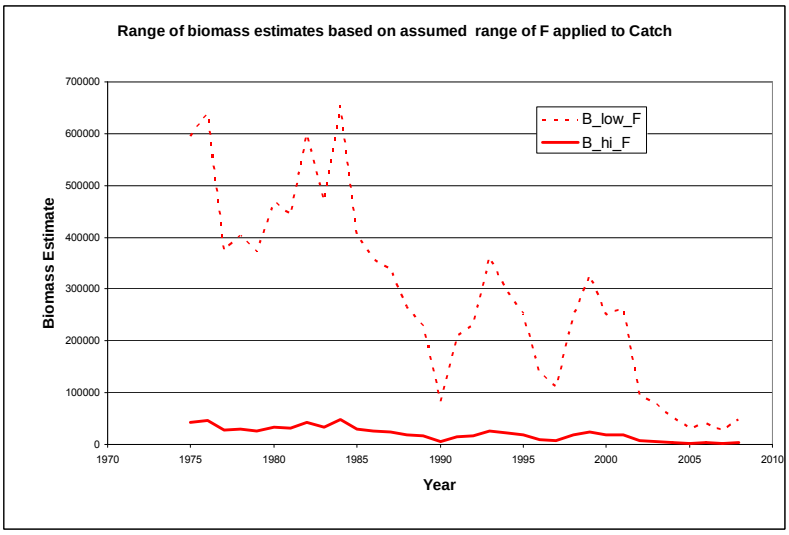
These bounds describe a set of feasible options that are consistent with the assumed ranges of q and F. In theory, a more sophisticated population model should lie within this feasible range.

Figure B.B1 illustrates the application of the envelope method using equations 1 to 4. Results suggest that biomasses necessary to support observed catches in the early 1980's were as high as 400,000 mt. Current population sizes since 2001 are likely to have been below 100,000 mt. The trend in minimum biomass estimates (high F, high q) is less pronounced but similar in relative trend. A comparison with biomass estimates from the final model run (Figure B.B2).

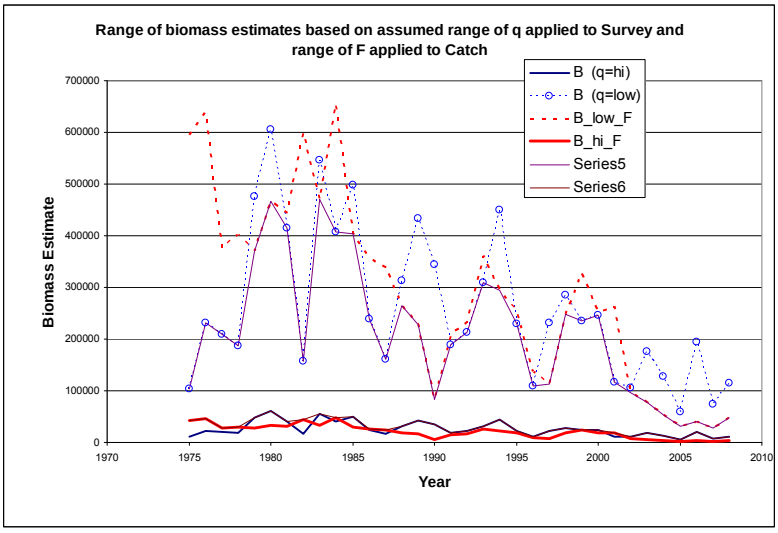
The envelope concept can also be extended to compute a range of feasible F values consistent with derived biomass estimates from Eq. 4. Assuming that $B_{1,t}$ and $B_{2,t}$ approximate average biomass at time t, then the ratio of C_t to $B_{1,t}$ or $B_{2,t}$ is a measure of biomass weighted F. These estimates can then be compared directly with the estimates of F from the KLAMZ model. Figure B.B3 suggests a comparable range of values except in 2003 to 2008. In these years the model-based estimate of F was about 0.03 which was lower than the lowest value of F (=0.05) used to construct the biomass series based on $B_{3,t}$.



A.



B.



C. Superposition of A and B with feasible set identified by Eq. 4.

Figure B4.1. Illustration of the envelope estimation method for the NEFSC fall survey index (A), and total catch (B). Panel C represents the feasible envelope of biomass estimates.

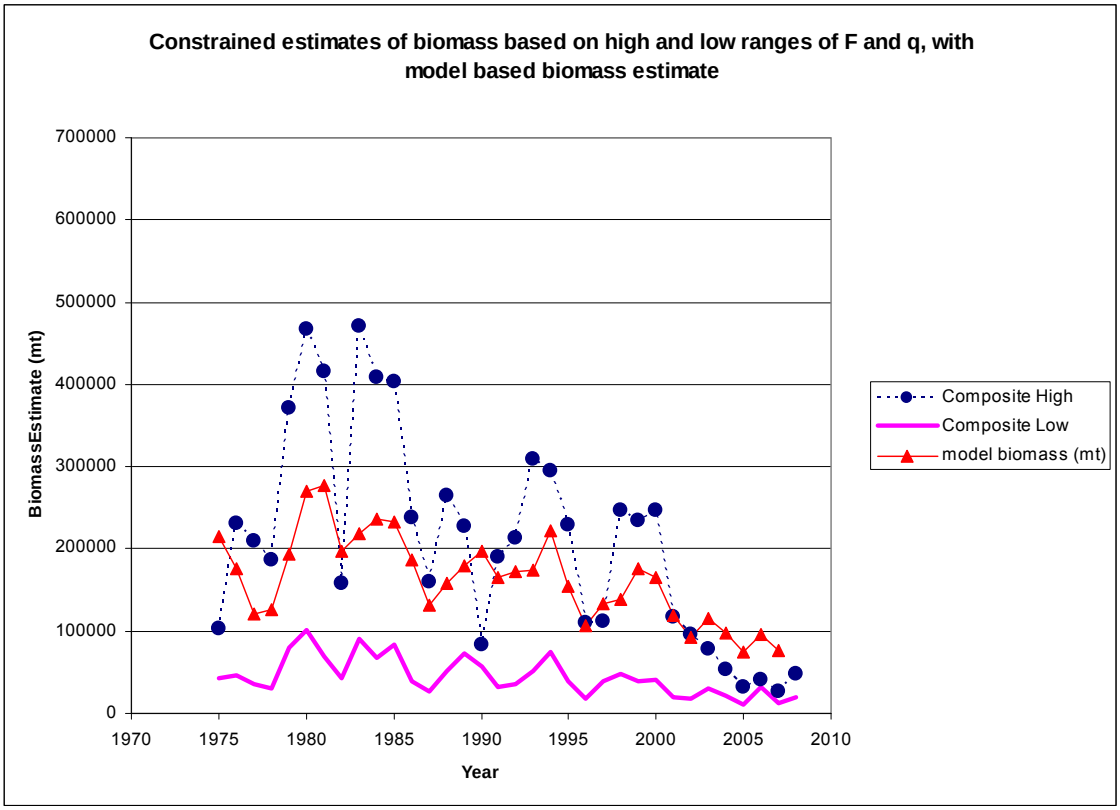


Figure B4.2. Comparison of the envelope measure of stock biomass with model based estimates.

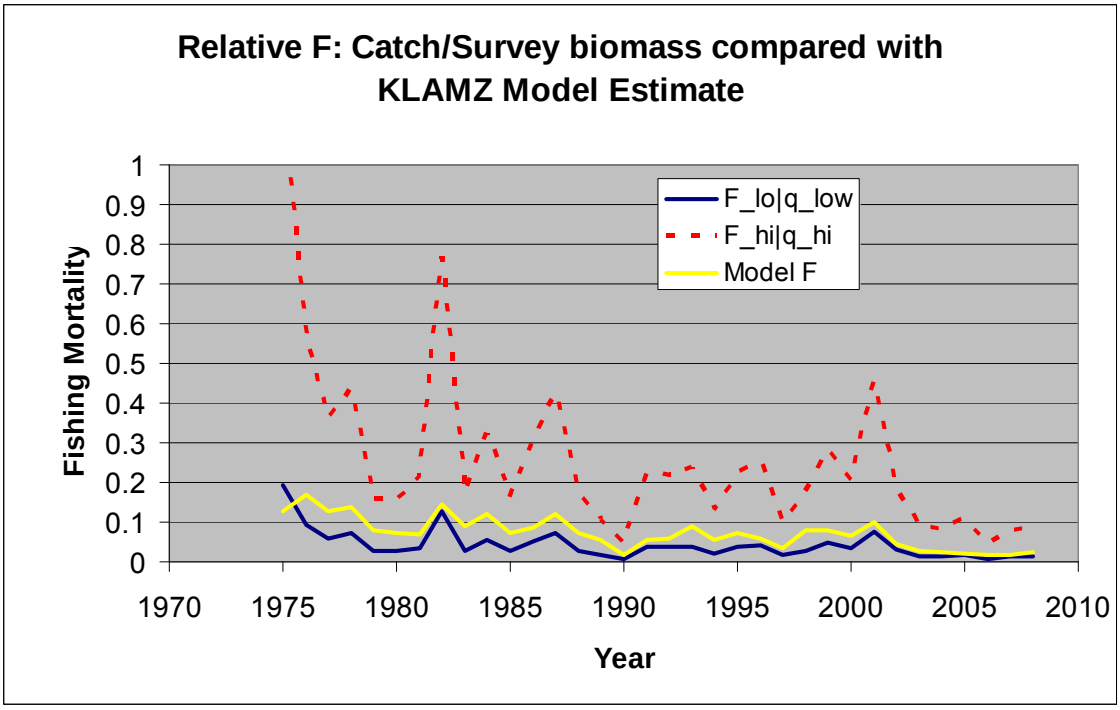


Figure B4.3. Comparison of KLAMZ estimate of fishing mortality with envelope derived from ratio of C_t to B_t derived from assumed range of q applied to survey indices.